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LETTER TO THE EDITOR

Long-wavelength fluctuations in the Ising spin glass

T Temesvári†, I Kondor† and C De Dominicis‡

† Institute for Theoretical Physics, Eötvös University, Budapest, Hungary
‡ Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay§, 91191 Gif-sur-Yvette Cedex, France

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Abstract. The overlaps of spin-spin correlations inside one phase space valley and between different valleys are calculated in the Gaussian approximation around Parisi's mean-field solution for the Ising spin glass. The large mass of the theory is shown to be related to the coherence length of the longitudinal fluctuations of the order parameter inside a single pure state. The long-wavelength asymptotics of correlation functions is worked out in detail and it is shown that an exact inequality between intra- and intervalley overlaps breaks down below dimension $d = 6$, signalling an internal inconsistency of the theory for $d < 6$.

By now we have a fairly complete picture of the infinitely long-range spin glass (see Mézard *et al* (1987) for a review). Results concerning Gaussian fluctuations about the mean-field solution have also become available (Sompolinsky and Zippelius 1983, Goltsev 1984, De Dominicis and Kondor 1984, 1985a), including the full set of Gaussian propagators for small external field h and reduced temperature $\tau = (T_c - T)/T_c$ (De Dominicis and Kondor 1985b, hereafter referred to as I). Although the formulae in I are, for $h, \tau \ll 1$, exact and given in closed forms, they are extremely complicated, which severely limits their possible use as building blocks of a future interacting field theory of spin glasses and also obscures their physical content. Our purpose here is to express these propagators in terms of overlaps of correlation functions inside and between phase space valleys, which gives them a clear physical meaning and allows us to set up an exact inequality between them. Through a detailed analysis of the long-wavelength behaviour we find that this inequality breaks down below $d = 6$ dimensions which is a signal of the inconsistency of the theory for $d < 6$.

We consider a standard Edwards and Anderson (1975) model with Ising spins and Gaussian distributed random couplings J_{ij} of finite range and assume that the dimension is high enough so that the phase space structure of the system is similar to that in mean-field theory (many valleys with an ultrametric organisation). Fluctuations can be characterised by various four-point functions, of which the simplest is the overlap of the spin-spin correlation function $\langle s_i s_k \rangle$ in valley α with that in valley β:

$$C_{\alpha\beta}(\boldsymbol{r}) = \frac{1}{N} \sum_i \langle s_i s_k \rangle_\alpha \langle s_i s_k \rangle_\beta = q_{\alpha\beta}^2 + \frac{1}{N} \sum_p \exp(-\mathrm{i}\boldsymbol{pr}) C_{\alpha\beta}(\boldsymbol{p}) \tag{1}$$

where $\boldsymbol{r} = \boldsymbol{r}_k - \boldsymbol{r}_i$ and $q_{\alpha\beta} = (1/N) \sum_i \langle s_i \rangle_\alpha \langle s_i \rangle_\beta$ is the usual overlap between the valleys. For $\alpha = \beta$ equation (1) describes correlations in a single valley.

§ Laboratoire de l'Institut de Recherche Fondamentale du Commissariat à l'Energie Atomique.

In principle, $C_{\alpha\beta}(\boldsymbol{r})$ could depend on the concrete realisation of the couplings J_{ij} and also on the valleys α, β. It can be shown, however, that $C_{\alpha\beta}(\boldsymbol{r})$ is self-averaging (i.e. independent of the sample J_{ij} in the thermodynamic limit) and depends on α, β only through the overlap $q_{\alpha\beta}$. In fact, the Fourier transform

$$C_{\alpha\beta}(\boldsymbol{p}) = \frac{1}{N}\sum_{ij} \exp[\mathrm{i}\boldsymbol{p}(\boldsymbol{r}_j - \boldsymbol{r}_i)]\langle s_i s_j\rangle_\alpha \langle s_i s_j\rangle_\beta - Nq^2_{\alpha\beta}\delta_{\boldsymbol{p},0} \tag{2}$$

can be calculated in the Gaussian approximation via the replica formalism and turns out to be the diagonal component of the propagator given in I. In the notation there:

$$C_{\alpha\beta}(\boldsymbol{p}) = G^{xx}_{11}(\boldsymbol{p}) \tag{3}$$

with $x = x(q_{\alpha\beta})$ and $x(q)$ is the inverse of Parisi's order parameter function.

The proof of the above statements proceeds by now standard lines first followed by Parisi (1983) and most clearly explained by Mézard and Virasoro (1985), and will, therefore, be omitted here. Similar statements can be made about the other overlaps:

$$\begin{aligned} C_{\alpha\beta\gamma}(\boldsymbol{r}) &= \frac{1}{N}\sum_{i} \langle s_i s_k\rangle_\alpha \langle s_i\rangle_\beta \langle s_k\rangle_\gamma \\ &= q_{\alpha\beta}q_{\alpha\gamma} + \frac{1}{N}\sum_{\boldsymbol{p}} \mathrm{e}^{-\mathrm{i}\boldsymbol{pr}} C_{\alpha\beta\gamma}(\boldsymbol{p}) \end{aligned} \tag{4}$$

and

$$\begin{aligned} C_{\alpha\beta\gamma\delta}(\boldsymbol{r}) &= \frac{1}{N}\sum_{i} \langle s_i\rangle_\alpha \langle s_i\rangle_\beta \langle s_k\rangle_\gamma \langle s_k\rangle_\delta \\ &= q_{\alpha\beta}q_{\gamma\delta} + \frac{1}{N}\sum_{\boldsymbol{p}} \mathrm{e}^{-\mathrm{i}\boldsymbol{pr}} C_{\alpha\beta\gamma\delta}(\boldsymbol{p}) \end{aligned} \tag{5}$$

which are also self-averaging and depend respectively on the pure states $\alpha\beta\gamma$ and $\alpha\beta\gamma\delta$ only through their mutual overlaps. The Fourier transforms in (4) and (5) are related to the off-diagonal components of the Gaussian propagator in I by

$$C_{\alpha\beta\gamma}(\boldsymbol{p}) = G^{xy}_{1z}(\boldsymbol{p}) \tag{6}$$

where $x = x(q_{\alpha\beta})$, $y = x(q_{\alpha\gamma})$ and $z = \max[x(q_{\beta\alpha}), x(q_{\beta\gamma})]$ and

$$C_{\alpha\beta\gamma\delta}(\boldsymbol{p}) = G^{xy}_{z_1 z_2}(\boldsymbol{p}) \tag{7}$$

with $x = x(q_{\alpha\beta})$, $y = x(q_{\gamma\delta})$, $z_1 = \max[x(q_{\alpha\gamma}), x(q_{\alpha\delta})]$, $z_2 = \max[x(q_{\beta\gamma}), x(q_{\beta\delta})]$, where ultrametricity makes at least two of the four variables x, y, z_1, z_2 coincide.

Equations (3), (6) and (7) give a clear physical meaning to the Gaussian propagators. In what follows we wish to make the closed expressions obtained for them in I more explicit. The discussion will be restricted to the case of zero field, $h = 0$. It is important to remember that the results in I are based on a truncated model, first proposed by Parisi (1979), valid only near T_c. This truncated model cannot be expected to produce results correctly beyond leading order in the reduced temperature τ. When investigating the long-wavelength behaviour of the propagators it is therefore important to expand the results in τ and keep only the leading terms.

There are two mass scales in the theory (De Dominicis and Kondor 1983): the band of large masses of order $m^2 \sim \tau \sim x_1$, with bandwidth $\sim\tau^2$, and that of the small masses of order $m^2 \sim \tau^2 \sim x_1^2$, reaching down to zero (x_1 is the breakpoint of Parisi's

$q(x)$). The infrared behaviour of the theory depends sensitively on the magnitude of the wavenumber p relative to these mass scales and also on the replica variables x, y, z_1, z_2, so there are a large number of regions. We have made a comprehensive study of all of them, but can present only a fraction of the results here.

Physically the most important case is when all replica variables go beyond the breakpoint x_1. In view of (1)-(7), this corresponds to overlaps of correlation functions inside a single valley, as first noted by Kotliar *et al* (1987). Now the propagators do not depend on the replica variables any more and we are left with only three different functions. Expanding the results in I to leading order in τ we find for these in the range $p^2 \gg x_1^2$ the following simple expressions:

$$G_{11}^{x_1x_1}(p) \equiv G_1(p) = \frac{1}{p^2}\left[1 + \frac{x_1}{p^2+x_1} + \left(\frac{x_1}{p^2+x_1}\right)^2\right] \tag{8a}$$

$$G_{1x_1}^{x_1x_1}(p) \equiv G_2(p) = \frac{1}{p^2}\left[\frac{1}{2}\frac{x_1}{p^2+x_1} + \left(\frac{x_1}{p^2+x_1}\right)^2\right] \tag{8b}$$

$$G_{x_1x_1}^{x_1x_1}(p) \equiv G_3(p) = \frac{1}{p^2}\left(\frac{x_1}{p^2+x_1}\right)^2. \tag{8c}$$

Since the masses form bands, $G_{1,2,3}$ should, in fact, display cuts. On the scales regarded here (i.e. for those p high above the small masses) these narrow bands, of width $x_1^2 \sim \tau^2$, are not resolved, however, and that is why we get the simple poles, one at zero, the other at the large mass, given in (8a)-(8c). It is all the more remarkable that if we work out $G_{1,2,3}$ in the other range, $p^2 \ll x_1^2$ (i.e. far below the upper band edge of the small masses) we find $3/p^2$, $3/2p^2$ and $1/p^2$, respectively, which precisely coincide with the long-wavelength limits of (8a)-(8c). To leading order in τ equations (8a)-(8c) can therefore be regarded as a good representation of $G_{1,2,3}$ both for $p^2 \gg x_1^2$ and for $p^2 \ll x_1^2$. Right around $p^2 \sim x_1^2$ there are corrections to (8a)-(8c) which, though explicitly known, are too complicated to be given here. The order of magnitude of $G_{1,2,3}$ for $p^2 \sim x_1^2$ is, however, still the same ($\sim 1/x_1^2$) as that given by (8a)-(8c) extrapolated to this range.

The fact that, in the particular case of the diagonal propagator G_{11}^{xx}, the strongest infrared power for $x > x_1$ is p^{-2} was already noted by De Dominicis and Kondor (1984).

Two particularly illuminating combinations of $G_{1,2,3}$ are the transverse or 'replicon' propagator (Bray and Moore 1987) $G_T = G_1 - 2G_2 + G_3$, and the longitudinal one $G_L = G_1 - 4G_2 + 3G_3$, related to the non-linear susceptibility (Chalupa 1977). For these we find

$$G_T = 1/p^2 \tag{9a}$$

$$G_L = 1/(p^2 + x_1) \qquad p^2 \gg x_1^2. \tag{9b}$$

The remarkably simple form for G_T is, as shown already in I, an identity, valid for all p and $T < T_c$. It is equally gratifying to see that, when expanded consistently, the very complicated formulae in I reduce to (9b), a most natural result within the Gaussian approximation, for the longitudinal propagator. The meaning of the large mass has now become clear: $\xi = x_1^{-1/2}$ is the coherence length of the longitudinal fluctuations of the order parameter in a single valley.

Nevertheless, (9b) requires some comments. For $p^2 \ll x_1$, G_L becomes much smaller than $G_{1,2,3}$. Actually, in the range of the small masses, $p^2 \sim x_1^2$, the leading terms in $G_{1,2,3}$ cancel exactly in the special combination G_L, which is therefore determined by the next corrections. Subleading terms are, however, not controlled reliably by the truncated model, so we are, in fact, unable to calculate G_L for $p^2 \lesssim x_1^2$. It may very well be that if we were able to go beyond the truncated model we could extend the range of the simple result for G_L also to $p^2 \ll x_1^2$, just as with $G_{1,2,3}$.

As we have already mentioned, some of the results above were already known. Perhaps the most important new ingredients we have added here are the precise values of the numerical coefficients and the scaling with x_1 in (8*a*)-(8*c*), which lead to the transparent result for G_L, and thereby to the interpretation of the large mass. Equations (8*a*)-(8*c*) and (9*a*, *b*), in conjunction with (1)-(7) providing their physical interpretation, given an (almost) full description of the long-wavelength Gaussian fluctuations in a single valley near T_c.

What remains to be understood is the behaviour in the range $p^2 \sim x_1^2$. The deviations from the simple results (8) and (9) observed there depend on the small masses whose meaning is at present unclear to us.

The range of dimensions where the many-valley picture is valid and where, accordingly, these results could represent at least a sensible zeroth-order approximation, is not known at the present time. (We make some remarks on this question at the end of this letter.) On the basis of numerical evidence, phenomenological scaling and renormalisation group ideas it has been argued (see Fisher and Huse (1988) for a most detailed exposition and Bray and Moore (1987) for a review of earlier work along these lines) that real, $d=3$, short-range spin glasses have a much simpler phase space structure, with (up to an overall reflection) a single pure state. If a meaningful comparison between these two theories, based on completely different physical pictures, can be made at all, it must be made between the phenomenological 'single-valley' theory and the *single-valley* predictions of the many-valley theory. In doing so one cannot help noticing some basic similarities: both theories predict a massless phase for all $T<T_c$, simple scaling forms for $G_{1,2,3}$ and G_T, and the cancellation of the leading singularity in G_L.

In addition to the single-valley results one also needs, for the complete characterisation of correlations in the many-valley picture, the intervalley overlaps of correlation functions. According to (1)-(7), intervalley overlaps correspond to some or all the replica variables going below the breakpoint x_1.

The simplest propagator (and the only one that we are able to discuss here in any detail) is the diagonal one, G_{11}^{xx}. Let us first assume that x is neither zero nor very close to x_1. Then we have two (overlapping) regions: $p^2 \gg x_1^2$ and $p^2 \ll x_1$. For $p^2 \gg x_1^2$ the propagator can be cast into the form

$$G_{11}^{xx} = \frac{1}{p^2} f\left(\frac{p^2}{x_1}; \frac{x}{x_1}\right) \tag{10a}$$

where f is a smooth scaling function with limiting behaviour

$$f = \begin{cases} 1 & \text{for } p^2 \gg x_1 \text{ and any } x \quad (10b) \\ \dfrac{x_1^2}{2p^4}\left(1-\dfrac{x^2}{x_1^2}\right)^2 & \text{for } p^2 \ll x_1. \quad (10c) \end{cases}$$

For $p^2 \ll x_1$ we have

$$G_{11}^{xx} = \frac{1}{p^4} g\left(\frac{p^2}{x_1^2}; \frac{x}{x_1}\right) \tag{11a}$$

with

$$g = \begin{cases} \dfrac{x_1^2}{2p^2}\left(1-\dfrac{x^2}{x_1^2}\right)^2 & \text{for } p^2 \gg x_1^2 \qquad (11b) \\ \dfrac{p}{x_1}\dfrac{x_1}{x} & \text{for } p^2 \ll x_1^2. \qquad (11c) \end{cases}$$

In the range of the overlap, $x_1^2 \ll p^2 \ll x_1$, (10*a*), (10*c*), (11*a*) and (11*b*) give, of course, the same result for G_{11}^{xx}. The essence of the above is that around and above the large masses the scaling power of G_{11}^{xx} is p^{-2} and p scales with the large mass; around and below the band edge of the small masses the scaling power is p^{-4} and p scales with the small mass; in between there is a smooth transition.

The scaling functions f and g are explicitly known but are too complicated to be recorded here.

Considering now valleys with zero overlap, i.e. $x=0$, we find

$$G_{11}^{00} = \begin{cases} \dfrac{\pi}{4p^4} & p^2 \ll x_1^2 \qquad (12a) \\ \dfrac{1}{p^2}\left[1+\dfrac{x_1}{p^2}+\dfrac{1}{2}\left(\dfrac{x_1}{p^2}\right)^2\right] & p^2 \gg x_1^2. \qquad (12b) \end{cases}$$

Equations (10*a*)-(10*c*) and (12*b*) are examples of a general finding: whenever p is much larger than the small-mass scale, the scaling power of G is p^{-2}, irrespective of the value of the replica variables. This means that the qualitative features of the infrared behaviour of the overlaps of correlation functions between two valleys are, except on the extreme long length scales, similar to those inside a single valley, even if the overlap of the local magnetisations is small.

Again, most of the qualitative features in (10)-(12) have been known for some years (see De Dominicis and Kondor (1984) and, for (12*a*), also Sompolinsky and Zippelius (1983)), but the values of the coefficients and the range of validity of the various infrared powers have not previously been published.

An interesting question is the overlap between two different but very close valleys, corresponding to $(x_1-x)/x_1 \ll 1$. Then $(x_1-x)^{1/2} \gg [x_1(x_1-x)]^{1/2} \gg (x_1-x)$ enter as new scales and the infrared behaviour becomes fairly complicated. We content ourselves by noting that, for $p \gg (x_1-x)^{1/2}$, G_{11}^{xx} goes over into $G_{11}^{x_1x_1}$, i.e. on length scales shorter than $(x_1-x)^{-1/2}$ the overlap of correlation functions between the two valleys is the same as inside a single valley, and it is only on scales longer than this distance that we can resolve them.

As for the off-diagonal propagators, they show so much detail that it is quite impossible to display any meaningful selection of the results. We stress, however, that we have made a thorough study of all the off-diagonal components and found the same general features as for G_{11}^{xx}: for $x_1^2 \ll p^2 \ll 1$, i.e. for p far above the small mass band, the dominant infrared power is always p^{-2}, while in the extreme long-wavelength limit, $p^2 \ll x_1^2$, the infrared singularity depends on the replica variables in a complicated manner, but there is no propagator whose Fourier transform would decay with distance slower than $1/r^{d-4}$.

This last statement contradicts a result by Goltsev (1986) who claims to have found a stronger p^{-6}-like divergence in one of the propagators (in our notation $G_{x_1x_1}^{x_1x_1}$) for which he finds $4x_1(1-x_1)/p^6$ instead of (8*c*). We checked his paper carefully and found general agreement up to his equation (20). At that point, with a quick reference to his equation (4), he gives a result for one of the components of the propagator (G^{yz}_{xx}, $x<y, z$ in our notation) which we claim is wrong, along with all its corollaries, including the power p^{-6}. Though he does not give sufficient details to allow us to pinpoint the error, its source can be guessed with a fair degree of certainty.

His equation (4), when written out in detail, is a set of seven coupled integral equations for the different components of the propagator. This set is not included in this paper, but had been published earlier in I. Now one may notice that one of the seven equations, equation (4) in I, has such a structure that the particular propagator G^{yz}_{xx}, $x<y, z$, can be isolated from the rest by taking the second derivative WRT one of the upper variables, say y. The resulting differential equation is trivially solved and yields $\sinh[(x_1-y)/p+y_0]$ for the y dependence of G^{yz}_{xx}, as in Goltsev's equation (21), with y_0 a constant. (Incidentally, there is an obvious misprint in the definition of y_0 as given below his equation (20): the correct expression is $y_0=\tanh^{-1}[p/(1-x_1)]$ instead of tanh as printed.) Now one may be tempted to invoke the symmetry of G^{yz}_{xx} in y and z (which is true) and conclude that G^{yz}_{xx} is of the form $F_2(x)\sinh[(x_1-y)/p+y_0]\sinh[(x_1-z)/p+y_0]$ which would be precisely Goltsev's equation (21). This conclusion is wrong, however. From the fact that even if a function G^{yz} is separable and symmetric in its variables it does not follow that it is of the form $G^{yz}=f(y)f(z)$. It may well be

$$G^{yz}=\begin{cases} f(y)g(z) & y<z \\ g(y)f(z) & z<y \end{cases} \tag{13}$$

in which case the information gained about f does not tell us anything about g. As it happens, G^{yz}_{xx}, not unusually for a Green function, depends on y and z like (13), but this can only be seen by studying, as in I, the full set of equations, instead of picking a single one of them.

Coming back to the discussion of the long-distance behaviour of the propagators we would like to draw attention to a strongly counterintuitive feature of our results: we invariably find the strongest infrared singularities for the smallest overlaps, which means that the slower the overlap of correlations between two valleys falls off with distance the further apart the valleys are in phase space. This is clearly paradoxical: the power-law-like falloff of overlaps is already a testimony of the strong correlation between the valleys, but two states that are far apart from the point of view of their local magnetisation distribution cannot have a larger overlap between their spin-spin correlation functions than the self-overlap of the correlation function inside a single state. In other words, for the overlaps defined in (1) the inequality

$$C_{\alpha\alpha}(\boldsymbol{r})>C_{\alpha\beta}(\boldsymbol{r}) \qquad \alpha\neq\beta \tag{14}$$

must evidently hold for any $\boldsymbol{r}$. According to (1), the $C_{\alpha\alpha}$ and $C_{\alpha\beta}$ in (14) are made up of the constants $q_{\alpha\alpha}$ and $q_{\alpha\beta}$, respectively, and the Fourier integrals. While the latter, with the infrared divergence getting stronger as $q_{\alpha\beta}$ decreases, would tend to reverse the inequality, this is overcompensated by the constant terms. Hence the resolution of the apparent paradox is that, though the overlap of correlations between distant valleys does indeed fall off more slowly, it falls off to a smaller constant, so in the end inequality (14) can be satisfied.

Whether it will actually be satisfied depends on the dimension, however. Assuming, for simplicity that x is not very close to x_1 it follows from the results in (8a) and (11a)

that in the range $r \sim 1/x_1$ equation (14) is roughly like

$$q(x_1)^2 + \frac{\text{constant}}{r^{d-2}} = \frac{x_1^2}{4} + \text{constant} \times x_1^{d-2} > q(x)^2 + \frac{\text{constant}}{r^{d-4}} = \frac{x^2}{4} + \text{constant} \times x_1^{d-4}. \tag{15}$$

As long as $d > 6$, this is always satisfied ($x < x_1 \ll 1$), but for $d < 6$ the last term, x_1^{d-4}, becomes the largest of all and the inequality is violated!

The breakdown of an obvious inequality can hardly be taken for anything but a signal that something goes seriously wrong with the many-valley picture in $d = 6$. In addition, this argument is not the only one that points to the special role of $d = 6$. The failure of locating a stable fixed point in the $\varepsilon = 6 - d$ expansion for the Almeida-Thouless transition (Bray and Roberts 1980) and other considerations led Moore and Bray (1985) to the suggestion that $d = 6$, usually believed to be the upper critical dimension for spin glasses, may, at the same time, be the *lower* critical dimension for the Parisi-type order, while an unpublished result by De Dominicis and Kondor (reviewed in part by Kondor (1985)) showed that the first loop correction to $q(x)$ becomes meaningless below $d = 6$. We do not believe that either or all of these arguments are strong enough to definitely rule out the possibility of some kind of many-valley structure below $d = 6$, but they are certainly strong enough to show that without a major reorganisation of the setup of the theory it will not be possible to continue it to $d < 6$. The observation about the breakdown of inequality (14) in $d = 6$ may be regarded as the most important result of the above investigation of the intervalley correlations.

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